

The NNLO QCD analysis of the CCFR data for xF_3 :
 Q^2 -dependence of the parameters.

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We continue the systematic next-to-next-to-leading order (NNLO) QCD analysis of the CCFR data for the xF_3 SF. The NNLO results for the Q^2 -dependence of the parameters, which describe the x -dependence of this SF, are presented.

1. Introduction

In the series of the subsequent papers[1]-[4] we concentrated ourselves on the fits of the CCFR data for xF_3 SF [5] at the LO, NLO and NNLO level with twist-4 effects included through the infrared-renormalon model of Refs.[6,7]. These fits were done using the Jacobi polynomial-Mellin moments version of the DGLAP method, developed and used in Refs.[8]- [10]. In the process of these fits we have taken into account the results of calculations of the NNLO corrections to the coefficient function of xF_3 SF [11] and the available analytical expressions for the NNLO corrections to the anomalous dimensions of the nonsinglet (NS) even moments with $n = 2, 4, 6, 8, 10$ [12], supplemented with the given in Ref.[1] $n = 3, 5, 7, 9$ similar numbers, obtained using the smooth interpolation procedure, which was previously proposed in Ref.[13].

The Jacobi polynomial method is based on the reconstruction of the SF from its moments $M_n^{NS} = \int_0^1 x^{n-1} F_3(x, Q^2) dx$ via the following equation:

$$xF_3^{N_{max}} = x^\alpha (1-x)^\beta \sum_{n=0}^{N_{max}} \Theta_n^{\alpha, \beta}(x) \sum_{j=0}^n c_j^{(n)}(\alpha, \beta) M_{j+2}^{NS}(Q^2) \quad (1)$$

where $n = 2, 3, \dots$, $c_j^{(n)}(\alpha, \beta)$ is the combination of Euler Γ -functions. For simplicity we present here the basic formula in the case when target mass corrections and higher-twist

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effects are neglected. More details can be found in Ref.[4], where the formulae for all these contributions are given.

Making the fits with the free Jacobi polynomial parameters α, β , we found that their values $\alpha \approx 0.7$, $\beta \approx 3$ correspond to the minimum in the plane (α, β) .

Using the standard model parameterization for the NS xF_3 SF, we are now defining the corresponding moments at the initial scale Q_0^2 :

$$M_n^{NS}(Q_0^2) = \int_0^1 x^{n-2} A(Q_0^2) x^{b(Q_0^2)} (1-x)^{c(Q_0^2)} (1 + \gamma(Q_0^2)x) dx \quad (2)$$

and evolve them to other transferred momentum using the solution of the renormalization-group QCD equations for the moments, which has the following form:

$$\frac{M_n^{NS}(Q^2)}{M_n^{NS}(Q_0^2)} = \left(\frac{A_s(Q^2)}{A_s(Q_0^2)} \right)^{\frac{\gamma_{NS}^{(0)}}{2\beta_0}} \frac{AD(n, Q^2) C^{(n)}(Q^2)}{AD(n, Q_0^2) C^{(n)}(Q_0^2)} \quad (3)$$

where $A_s(Q^2) = \alpha_s(Q^2)/(4\pi)$ is the \overline{MS} -scheme expansion parameter, $AD(n, Q^2) = 1 + p(n)A_s(Q^2) + q(n)A_s(Q^2)^2 + \dots$ comes from the expansion of the anomalous dimensions term of the solution of the corresponding renormalization-group equation, $C^{(n)}(Q^2) = 1 + C^{(1)}(n)A_s(Q^2) + C^{(2)}(n)A_s(Q^2)^2 + \dots$ is the coefficient function of the n -th moment of the xF_3 SF. The numerical values of the coefficients $p(n)$, $q(n)$, $C^{(1)}(n)$ and $C^{(2)}(n)$, normalized to the case of $f = 4$ numbers of active flavours, are given in Ref.[3,4]. In our fits we will use well-known LO, NLO and NNLO expressions of α_s , written down as the expansions in the inverse powers of $\ln(Q^2/\Lambda_{\overline{MS}}^{(4)})^2$ -terms in the LO, NLO and NNLO.

2. Presentation of the results

Here we complete our previous analysis of the CCFR'97 data of Refs.[2–4] and concentrate our attention on the extraction of the Q^2 dependence of the parameters of the model of Eq.(2) for xF_3 SF (which, of course, are related to the parton distributions parameters). In Table 1 we present the results of our fits of the CCFR xF_3 data with the cut $Q^2 > 5 \text{ GeV}^2$ for different fixed values of Q_0^2 with TMC taken into account, but twist-4 contributions neglected. Several comments are in order:

1) For the x -shape parameters the shift between LO and NLO values is larger than between NLO and NNLO results (except parameter γ). This fact indicates the suppression of the uncertainties of the values of the parameters as the result of taking into account higher order perturbative QCD effects.

2) The numerical values of the parameters, obtained in the fits at the NNLO, are in a good agreement with those obtained in Ref. [14] where HT corrections were taken into account and the nuclear effects parameterized by deuteron model approximation of xF_3^{Fe} [15].

3) The effect of the increase of the values of the parameter c and the simultaneous decrease of the values of the parameter b , which is observed while increasing the value of Q_0^2 -scale, is in a qualitative agreement with the theoretical predictions of the works of Ref.[16] and Ref.[17].

4) The value of the parameter b , which governs the low- x behaviour of the xF_3 SF, is in agreement with the recent theoretical calculation of the low- x behaviour of the NS structure functions [18].

Table 1

The Q_0^2 -dependence of the parameters of xF_3 model, extracted from the fits of CCFR'97 data with the cut $Q^2 > 5 \text{ GeV}^2$. The statistical errors are taken into account.

$Q_0^2 = 5 \text{ GeV}^2$	$\Lambda_{\overline{MS}}^{(4)} [\text{MeV}]$	A	b	c	γ	$\chi^2/\text{n.e.p.}$
LO	266 ± 35	5.13 ± 0.46	0.72 ± 0.03	3.87 ± 0.05	1.42 ± 0.33	113.2/86
NLO	341 ± 30	4.05 ± 0.36	0.65 ± 0.03	3.71 ± 0.06	1.93 ± 0.36	87.1/86
NNLO	293 ± 29	4.25 ± 0.38	0.66 ± 0.03	3.56 ± 0.07	1.33 ± 0.33	78.4/86
$Q_0^2 = 8 \text{ GeV}^2$						
LO	266 ± 35	5.10 ± 0.15	0.70 ± 0.01	3.94 ± 0.04	1.23 ± 0.12	113.2/86
NLO	340 ± 40	4.38 ± 0.17	0.66 ± 0.02	3.81 ± 0.07	1.47 ± 0.19	87.4/86
NNLO	312 ± 33	4.42 ± 0.36	0.66 ± 0.03	3.68 ± 0.07	1.13 ± 0.31	76.5/86
$Q_0^2 = 10 \text{ GeV}^2$						
LO	265 ± 34	5.07 ± 0.15	0.70 ± 0.01	3.97 ± 0.04	1.16 ± 0.12	113.2/86
NLO	340 ± 35	4.48 ± 0.15	0.66 ± 0.02	3.85 ± 0.04	1.32 ± 0.10	87.5/86
NNLO	318 ± 33	4.50 ± 0.36	0.65 ± 0.03	3.73 ± 0.07	1.05 ± 0.31	76.3/86
$Q_0^2 = 15 \text{ GeV}^2$						
LO	265 ± 35	5.02 ± 0.16	0.68 ± 0.01	4.01 ± 0.04	1.04 ± 0.11	113.1/86
NLO	339 ± 37	4.61 ± 0.42	0.65 ± 0.03	3.92 ± 0.11	1.08 ± 0.42	87.6/86
NNLO	324 ± 34	4.60 ± 0.34	0.65 ± 0.03	3.83 ± 0.07	0.92 ± 0.29	76.6/86
$Q_0^2 = 20 \text{ GeV}^2$						
LO	264 ± 35	4.98 ± 0.15	0.68 ± 0.01	4.05 ± 0.04	0.96 ± 0.12	113.1/86
NLO	339 ± 36	4.67 ± 0.16	0.65 ± 0.02	3.96 ± 0.05	0.95 ± 0.13	87.6/76
NNLO	326 ± 35	4.70 ± 0.34	0.65 ± 0.03	3.88 ± 0.08	0.80 ± 0.30	77.0/86
$Q_0^2 = 30 \text{ GeV}^2$						
LO	264 ± 37	4.92 ± 0.17	0.67 ± 0.02	4.09 ± 0.05	0.86 ± 0.09	113.0/86
NLO	338 ± 37	4.72 ± 0.43	0.64 ± 0.03	4.01 ± 0.08	0.80 ± 0.35	87.5/86
NNLO	327 ± 35	4.78 ± 0.32	0.64 ± 0.03	3.96 ± 0.08	0.67 ± 0.27	77.8/86
$Q_0^2 = 50 \text{ GeV}^2$						
LO	264 ± 35	4.84 ± 0.15	0.65 ± 0.01	4.13 ± 0.04	0.75 ± 0.12	112.9/86
NLO	337 ± 34	4.74 ± 0.13	0.64 ± 0.01	4.06 ± 0.06	0.64 ± 0.14	87.5/86
NNLO	326 ± 36	4.85 ± 0.31	0.64 ± 0.02	4.03 ± 0.09	0.53 ± 0.27	78.8/86
$Q_0^2 = 100 \text{ GeV}^2$						
LO	263 ± 36	4.73 ± 0.23	0.64 ± 0.02	4.19 ± 0.09	0.62 ± 0.24	112.6/86
NLO	337 ± 37	4.73 ± 0.15	0.62 ± 0.02	4.12 ± 0.06	0.46 ± 0.12	87.3/86
NNLO	325 ± 36	4.91 ± 0.28	0.63 ± 0.02	4.11 ± 0.10	0.36 ± 0.25	80.0/86

5) In the case of $Q_0^2 = 5 \text{ GeV}^2$ the NLO results for the parameters b and c are in agreement with the outcomes of the NLO fits of the CCFR'97 data, which were made in Ref.[19] with the help of the standard realization of the DGLAP equation. Note, however, that the value of parameter γ , obtained in Ref.[19], turned out to be comparable with zero. It is of interest to trace the origin of this only difference with the presented in Table 1 results.

6) One can see that the values of $\Lambda_{\overline{MS}}^{(4)}$, which come from the LO and NLO fits, are rather stable to the choice of the initial scale Q_0^2 . However, the results of the NNLO fits are more sensitive to its variation. Indeed, despite the fact that for smaller values of Q_0^2 the difference between NLO and NNLO values of the parameters A , b , c , γ are not large, the NNLO values of $\Lambda_{\overline{MS}}^{(4)}$ are considerably smaller, than the NLO ones. This effect is decreasing while increasing Q_0^2 from 5 GeV^2 up to 20 GeV^2 . For $Q^2 \geq 20 \text{ GeV}^2$ the NNLO values of $\Lambda_{\overline{MS}}^{(4)}$ are becoming rather stable.

7) We think that this effects is related to rather peculiar behavior of the NNLO perturbative QCD expansion of $n = 2$ moment. Indeed, taking into account the exact numerical values of the coefficients $p(2)$, $q(2)$, $C^{(1)}(2)$ and $C^{(2)}(2)$, given in Refs.[3,4], we find that the perturbative behavior of the $n = 2$ moment is determined by the following series:

$$AD(2, Q^2)C^{(2)}(Q^2) = 1 - 0.132A_s(Q^2) - 46.16\left(A_s(Q^2)\right)^2 + \dots \quad (4)$$

where the value of the relatively large A_s^2 coefficient mainly comes from the NNLO term of the coefficient function of $n = 2$ moment. Thus we conclude, that it is safer to start the QCD evolution from the scale $Q_0^2 = 20 \text{ GeV}^2$, where the numerical value of the A_s^2 contribution in Eq.(4) is smaller, and therefore the complicated asymptotic structure of the perturbative expansion of $n = 2$ moment is not yet manifesting itself. Note, that the choice of this initial scale is also empirically supported by the fact that it is lying in the mid of the kinematic region of the CCFR data.

8) It is interesting to note, that the nonstandard structure of the perturbative QCD expression for Eq.(4) is manifesting itself not only in the case of $f = 4$ number of flavours. Indeed, in the cases of $f = 3$ and $f = 5$ the expressions for the NNLO approximations of the corresponding solutions of the renormalization-group equations read

$$\begin{aligned} AD(2, Q^2)C^{(2)}(Q^2) &= 1 - 0.271A_s(Q^2) - 43.51\left(A_s(Q^2)\right)^2 + \dots \\ AD(2, Q^2)C^{(2)}(Q^2) &= 1 + 0.126A_s(Q^2) - 48.88\left(A_s(Q^2)\right)^2 + \dots \end{aligned} \quad (5)$$

Thus, we conclude, that the unnatural behaviour of the order $O(A_s^2)$ approximation of the renormalization-group improved Mellin moment cannot be avoided after changing the number of active flavours.

In Table 2 we present the results of our fits of the CCFR'97 data including target mass corrections both without twist-4 corrections and with twist-4 contributions, fixed through the IRR model of Ref.[6] as

$$M_n^{IRR} = \tilde{C}(n)M_n^{NS}(Q^2)\frac{A_2'}{Q^2} \quad (6)$$

Table 2

The results of the fits of CCFR'97 data with the cut $Q^2 > 5 \text{ GeV}^2$, obtained in the case $Q_0^2 = 20 \text{ GeV}^2$. N³LO means the application of [0/2] expanded Padé approximants.

	$\Lambda_{\overline{MS}}^{(4)} [\text{MeV}]$	A	b	c	γ	$A_2' [\text{GeV}^2]$	$\chi^2/\text{n.e.p.}$
LO	264 ± 36	4.98 ± 0.23	0.68 ± 0.02	4.05 ± 0.05	0.96 ± 0.18	—	113.1/86
	433 ± 51	4.69 ± 0.13	0.64 ± 0.01	4.03 ± 0.04	1.16 ± 0.12	-0.33 ± 0.12	83.1/86
NLO	339 ± 35	4.67 ± 0.11	0.65 ± 0.01	3.96 ± 0.04	0.95 ± 0.09	—	87.6/86
	369 ± 37	4.62 ± 0.16	0.64 ± 0.01	3.95 ± 0.05	0.98 ± 0.17	-0.12 ± 0.06	82.3/86
NNLO	326 ± 35	4.70 ± 0.34	0.65 ± 0.03	3.88 ± 0.08	0.80 ± 0.28	—	77.0/86
	327 ± 35	4.70 ± 0.34	0.65 ± 0.03	3.88 ± 0.08	0.80 ± 0.29	-0.01 ± 0.05	76.9/86
N ³ LO	335 ± 37	4.77 ± 0.34	0.65 ± 0.03	3.85 ± 0.08	0.71 ± 0.28	—	77.9/86
(Padé)	340 ± 37	4.78 ± 0.34	0.65 ± 0.03	3.85 ± 0.08	0.71 ± 0.28	-0.04 ± 0.05	77.2/86

where $\tilde{C}(n) = -n - 4 + 2/(n+1) + 4/((n+2) + 4S_1(n))$ ($S_1(n) = \sum_{j=1}^n 1/j$) and A_2' is the free parameter.

The theoretical uncertainties of the NNLO results are probed by using the [0/2] Padé expanded approximants in Eq.(3) and using the N³LO expression for the expansion parameter A_s , which depends from the calculated recently four-loop coefficient of the QCD β -function [20].

The problem of comparison of the results of the fits with the IRR-model estimates and the effect of decreasing the value of its parameter A_2' at the NNLO was analysed in detail in Ref.[2] and Ref.[4] in particular. Here we would like to draw the attention to the fact, that besides the NLO value of $\Lambda_{\overline{MS}}^{(4)}$ (and thus α_s) is sensitive to the twist-4 contributions, modeled with the help of the IRR approach, the parameters of the x -dependence of the xF_3 SF (and thus parton distribution parameters) are more sensitive to the choice of Q_0^2 -scale, than to the twist-4 effects (see Tables 1,2 above and Table 5 in Ref.[4]). Taking into account this effects might allow to fix theoretical uncertainties of the parton distributions parameters. The importance of this theoretical problem was emphasized in Ref.[21].

To conclude, let us present the NLO and NNLO values of $\alpha_s(M_Z)$, obtained in Ref.[4] from the results of the fits of CCFR'97 data for xF_3 with twist-4 terms modeled through the IRR approach (see Table 2) and using the \overline{MS} -matching condition in different orders of perturbation theory [22–24]:

$$\begin{aligned}
\text{NLO } \alpha_s(M_Z) &= 0.120 \pm 0.003(\text{stat}) \pm 0.005(\text{syst}) \pm 0.004(\text{theor}) \\
\text{NNLO } \alpha_s(M_Z) &= 0.118 \pm 0.003(\text{stat}) \pm 0.005(\text{syst}) \pm 0.003(\text{theor})
\end{aligned} \tag{7}$$

The systematical uncertainty in these results are determined by using the original CCFR considerations and the theoretical errors are fixed using the results of [0/2] Padé approximations fits and the proposals of Ref.[25,26] to estimate the ambiguities due to smooth transition to the world with $f = 5$ numbers of flavours. It should be noted that the NLO result is in agreement with the one, obtained in Ref.[19] using the DGLAP equation and taking into consideration the correlations of statistical and systematical uncertainties.

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